AP Calculus BC

Unit 10
Convergence and Divergence Tests

AP Calculus BC – Worksheet 79 **Direct Comparison and Limit Comparison Tests**

Use the Direct Comparison Test to show that the series either converge or diverge.

1) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$	2) $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$	3) $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$
4) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$	5) $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$	6) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$

Use the Limit Comparison Test to show that the series either converge or diverge.

7) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$	8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$	9) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$
10) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$	11) $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$	12) $\sum_{n=3}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$

What is the value of $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$?

(A) -2 (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 3 (E) The series diverges.

For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

AP Calculus BC – Worksheet 81 **Ratio Test, Root Test, and Review**

*Optional problems

Determine if the following series converge of diverge using the indicated test.

Ratio Test: 1. $\sum \frac{3^k}{k!}$ 2. $\sum \frac{4^k}{k^2}$ 3. $\sum \frac{k!}{k^3}$

Root Test: 4. $\sum \left(\frac{4n-5}{2n+1}\right)^n$ 5. $\sum \frac{k}{5^k}$

Use any appropriate test to determine if the series converge or diverge.

6. $\sum \frac{2^k}{k^3}$ 7. $\sum \frac{7^k}{k!}$ 8. $\sum \frac{k^2}{5^k}$ 9. $\sum k^{50}e^{-k}$ *10. $\sum \frac{\cos n\pi}{\sqrt{n}}$

11. $\sum \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$ 12. $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3}$ 13. $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$ *14. $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^{3/2}}$

$\int_1^{\infty} \frac{1}{x^p}dx$ and $\int_0^1 \frac{1}{x^p}dx$ both diverge when $p =$

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) –1

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ II. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ III. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

(A) None (B) II only (C) III only (D) I and II only (E) II and III only

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Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

AP Calculus BC – Worksheet 82 **Alternating Series Test**

Which of the alternating series in Exercises 1–10 converge, and which diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$

3. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{10}\right)^n$

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{10^n}{n^{10}}$

5. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\ln n}$

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

7. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln n}{\ln n^2}$

8. $\sum_{n=1}^{\infty}(-1)^{n}\ln\left(1+\frac{1}{n}\right)$

9. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

10. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3\sqrt{n+1}}{\sqrt{n}+1}$

Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

11. $\sum_{n=1}^{\infty}(-1)^{n+1}(0.1)^n$

12. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

13. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\sqrt{n}}$

14. $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$

15. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

Which of the following statements are true about the series $\sum_{n=2}^{\infty}a_n$, where $a_n=\frac{(-1)^n}{\sqrt{n}+(-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}|\le|a_n|$ for all $n\ge 2$

III. $\lim_{n\to\infty}a_n=0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

AP Calculus BC – Worksheet 83 **Absolute Convergence**

Which of the series in exercises 1-8 converge absolutely, which converge, and which diverge?

1) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{10}\right)^n$	2) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$
3) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$	4) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$
5) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+3}$	6) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n}$
7) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$	8) $\sum_{n=1}^{\infty}\frac{(-1)^n(n+1)^n}{(2n)^n}$

Which of the following series converge, and which diverge?

9) $\sum_{n=1}^{\infty}\frac{n^{10}}{10^n}$	10) $\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$
11) $\sum_{n=1}^{\infty}\frac{(n+1)(n+2)}{n!}$	12) $\sum_{n=1}^{\infty}\frac{n2^n(n+1)!}{3^n n!}$

The infinite series $\sum_{k=1}^{\infty}a_k$ has nth partial sum $S_n=\frac{n}{3n+1}$ for $n\ge 1$. What is the sum of the series $\sum_{k=1}^{\infty}a_k$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) The series diverges.

The infinite series $\sum_{k=1}^{\infty}a_k$ has nth partial sum $S_n=(-1)^{n+1}$ for $n\ge 1$. What is the sum of the series $\sum_{k=1}^{\infty}a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

AP Calculus BC – Worksheet 84 **Convergence and Divergence of Series**

Determine if the following series converge absolutely, converge conditionally, or diverge. State which test is used.

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	2) $\sum_{n=1}^{\infty} \frac{-5}{n}$	3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
4) $\sum_{n=1}^{\infty} \frac{1}{2n^3}$	5) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$	6) $\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$
7) $\sum_{n=1}^{\infty} \frac{n+1}{n!}$	8) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$	9) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$
10) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2+1}}$	11) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1}$	12) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ II. $\sum_{n=1}^{\infty} e^{-n}$ III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$ (B) $|a_n| < 1$ for all n (C) $\sum_{n=1}^{\infty} a_n = 0$ (D) $\sum_{n=1}^{\infty} n a_n$ diverges. (E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

AP Calculus BC – Worksheet 85 **Convergence and Divergence**

1. The sum of the infinite geometric series $\frac{2}{5}+\frac{6}{20}+\frac{18}{80}+\frac{54}{320}+\ldots$ is

   A) $\frac{8}{15}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{5}{4}$ E) $\frac{8}{5}$

2. Which of the following series converge? I. $\sum_{n=1}^{\infty}\frac{3}{n}$ II. $\sum_{n=1}^{\infty}\frac{n+1}{n+4}$ III. $\sum_{n=1}^{\infty}\frac{-2}{(-5)^n}$

   A) I only B) I and II only C) I and III only D) II only E) III only

3 – 8: Determine if the following converge or diverge, state the test used and show all work.

3. $\sum_{n=1}^{\infty}\frac{5n-2}{n\,(3^n)}$ 4. $\sum_{n=1}^{\infty}\frac{2n}{3n^2+1}$ 5. $\sum_{n=1}^{\infty}\frac{5^n}{3n^2+1}$

6. $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ 7. $\sum_{n=1}^{\infty}\frac{\sin n}{3^n}$ 8. $\sum_{n=1}^{\infty}\frac{n\,(3^n)}{n!}$

9. Show that the series $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$ converges by using the integral test.

10. Show that the series $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$ converges by using the comparison test.

11. Converge (conditionally/absolutely) or diverge: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3n}$

12. Converge (conditionally/absolutely) or diverge: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n+3}{3n^2+4}$

AP Calculus BC – Worksheet 86 **Series Convergence and Divergence**

Show which test was used to find whether each series converges or diverges.

1	Find $\sum_{n=1}^{\infty}\left(\frac{8}{9}\right)^n$
2	Which of these series **converges**? I. $\sum_{n=1}^{\infty}\frac{\cos n}{2^n}$ II. $\sum_{n=1}^{\infty}\frac{1}{\left(\sqrt{5}+1\right)^n}$ III. $\sum_{n=1}^{\infty}\frac{2+\sin n}{n}$
3	Consider the series $\sum_{n=1}^{\infty}\frac{n^n}{n!}$. Use the ratio test to determine if it converges or diverges.
4	The integral test confirms that the series $\sum_{n=1}^{\infty}\left(\frac{e^{\frac{1}{n}}}{n^2}\right)$ converges. What is $\int_1^{\infty}\frac{e^{\frac{1}{x}}}{x^2}dx$?
5	**Multiple Choice** Which of these series are divergent? Show work or explain which test was used for each series. a) $\sum_{n=1}^{\infty}\frac{1}{2^n}$ b) $\sum_{n=1}^{\infty}\frac{1}{n!}$ c) $\sum_{n=1}^{\infty}\frac{1}{n}$ d) $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ e) $\sum_{n=1}^{\infty}\frac{1}{n^2}$
6	**Multiple Choice** Which of the following series converges? Show work or explain which test was used for each series. a) $\sum_{n=1}^{\infty}\frac{1}{\ln n}$ b) $\sum_{n=1}^{\infty}\frac{1}{n}$ c) $\sum_{n=1}^{\infty}\frac{1}{e^n}$ d) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ e) $\sum_{n=1}^{\infty}n$
7	Show all work for each and determine whether the following series converge or diverge. a) $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ b) $\sum_{n=1}^{\infty}\frac{n+1}{n+2}$ c) $\sum_{n=1}^{\infty}\frac{3^n}{n!}$ d) $\sum_{n=1}^{\infty}\frac{n^2}{n^3+1}$ e) $\sum_{n=1}^{\infty}\frac{n}{2^n}$ f) $\sum_{n=1}^{\infty}\frac{\sin n}{3^n}$ g) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{2n^2+3}}$ h) $\sum_{n=1}^{\infty}\frac{2n}{n^2-1}$
8	Does the following converge conditionally or absolutely or does it diverge? $\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^3+1}$

Answers:

1. 8 2. I and II 3. Diverges 4. -1+e 5. C 6. C

7. a) Div b) Div c) Conv d) Div e) Conv f) Conv g) Div h) Div

8. Converges Conditionally